GRAVITY CURRENTS IN LINEARLY STRATIFIED AMBIENT FLUID

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KEY POINTS:

- *The present work aims to study the behavior of a dense fluid current in a rectangular (or in a circular) cross-section channel. The denser fluid propagates into a less dense ambient fluid, with linear stratification of density in the vertical*
- *A simplified model was used to compute the front speed of the gravity current. The theoretical speed was obtained by: a) imposing the continuity between the injected and the propagating fluid; b) by considering the jump condition and the energetic balance at the head of the current.*
- *Seventy-six experiments were performed. The results show a systematic overestimation of the experimental front speed. The discrepancy decreases significantly for increasing value of the parameter* S *(representative of the relation between the density stratification of the ambient fluid and the density of the intruding current) and is at a minimum for $S \to 1$ (i.e., density of the intruding current equal to the bottom density of the ambient fluid).*

1 INTRODUCTION

A gravity current (GC) occurs when a denser fluid with mass density ρ_c propagates into a less dense fluid, called ambient fluid, and the propagation is mainly in the horizontal direction. Generally speaking, the ambient fluid can be homogeneous with mass density ρ_a, or stratified with minimum density ρ_0 at the top and maximum density ρ_b at the bottom, and a generic shape f the density profile. This kind of current (in both homogeneous and stratified ambient) can be found in different natural phenomena, such as avalanches, pyroclastic and lava flows, and several industrial activities.

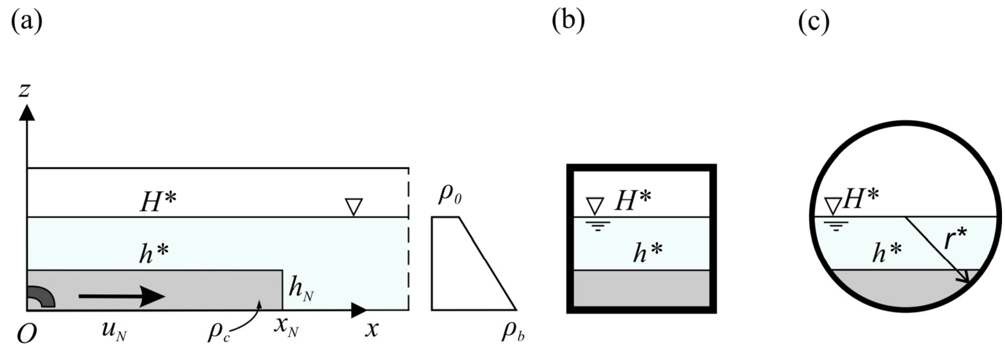

Figure 1. A schematic description of the simplified model: (a) shows a side-view of the gravity current with thickness $h^*(x)$, (b) shows the cross-section of the rectangular channel, with H^* = 11 cm, and panel (c) illustrates the cross-section of the circular channel, with $H^* = r^*$.

The front speed was computed on the base of the Benjamin condition extended to a linearly stratified ambient, see *Ungarish* (2006). The front velocity was computed by using the model described in *Shringarpure et al.* (2013) for gravity currents with a constant inflow, and according to *Ungarish* (2013) for the circular cross-section. The layout of the model is shown in Fig.1.

2 THEORETICAL MODEL

Here steady-state high Reynolds number gravity currents (GC) in a linearly density stratified ambient fluid are considered. The currents are of the Boussinesq type, i.e., $\rho_c/\rho_0 \approx 1$ where ρ_c is the density of the current and ρ_0 is the minimum density of the ambient fluid. One of the goals is the comparison between the theoretical and experimental values of the front speed u_N. The theoretical model considers the continuity equation for the incompressible fluid of the current

$$\frac{\partial A}{\partial t}+\frac{\partial uA}{\partial x}=0 \tag{1}$$

where $A(h(x,t))$ is the cross-section area occupied by the current and $u(x,t)$ is the velocity averaged over A. The explicit form of A as a function of h depends on the shape of the cross-section.

The second equation is given by the momentum balance in the direction of the current propagation, with neglected viscosity and with compensation between inertia and buoyancy:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-g'\left(1-S+S\frac{h}{H}\right)\frac{\partial h}{\partial x} \tag{2}$$

where $g' = \varepsilon g$, $\varepsilon = (\rho_c - \rho_0) / \rho_0$ and $S = (\rho_b - \rho_0)/ (\rho_c - \rho_0)$ represents the linear-stratification effect. The formulation requires a boundary condition at the front, treated as a discontinuity. As an approximation, it is assumed that for a stratified ambient fluid the rigorous analysis in *Ungarish* (2012) can be extended with the following approximation

$$u_N = Fr\ h_N^{1/2}\Psi^{1/2},\quad \Psi = 1-S\left[1-\frac{1}{2}\frac{h_N}{H*}(1+\gamma)\right], \tag{3}$$

where $Fr = Fr(h_N)$ is the Froude number, Ψ is a stratification coefficient with γ dependent on the cross section shape and the variables are made dimensionless by assuming the length scale equal to the ambient fluid thickness $H*$. The velocity scale is $U* = (g' H*)^{1/2}$; the time scale is $T* = H* / U*$. The discharge is scaled by $U*H*^2$ and the Reynolds number of the current is $Re = U*H* / \nu_c$. The second boundary condition at the inlet section depends on the kind of flow. For the lock-release problem the condition is $u = 0$ and $h = h_0$ at $t = 0$. For the constant influx problem the Froude number at the inlet is assumed constant over the duration of the simulation (see *Shringapure et al.* (2013)). The stratification of the ambient fluid reduces the front speed with respect to the homogeneous case.

3 MATERIALS AND METHODS

The tests were carried out in the Hydraulic Laboratory of DICATeA (University of Parma). The experimental facilities were (i) a circular cross-section channel with internal radius $r* = 9.5$ cm and length $l = 605$ cm (Fig.1c), and (ii) a rectangular cross-section channel 14 x 14 cm^2 (Fig.1b). The stratification of the ambient fluid was obtained by adopting the technique detailed in *Hill* (2002), injecting the fluid at the bottom of the tank. The density stratification in the tank was tested using a syringe attached to a needle whose tip is moved in the vertical at different level. The density of the drawn liquid was measured with a glass floating densitometer with accuracy equal to 1 kg/m^3.

In the constant influx tests a centrifugal pump, controlled by a DAQ through a PID feedback system was used to inject the denser fluid with a constant known discharge, accurate within 1% of the instantaneous value. At the opposite end section of the channel (with respect to the injection section) instead of the weir used for similar experiments in *Longo et al.* (2016), an aspiration system controlled in feedback with an Ultrasonic distance meter was installed to guarantee a constant water level during the whole test. The fluid was pumped out by several small tubes at different heights, in order to preserve the ambient fluid stratification and to avoid the selective withdrawal. hence a no-return flow configuration was adopted.

The lock release experiments were carried out only in the circular cross section tank, with a lock 100 cm long and a total length of the channel equal to 500 cm. For these experiments there was no outflow and the return-flow condition was satisfied.

The ambient fluid was linearly stratified with $S \in [0, 1]$ and in all tests the intruding current was salt water with density $\rho_c = 1100$ kg/m^3, added with Aniline dye for an easy visualization. A grid was stuck at the bottom of the channels to measure the front position of the intruding current. The front of the propagating current was recorded by a full HD video camera, with data rate of 25 f.p.s. Afterwards the front position was measured by analyzing the frames.

In all tests with constant influx the height of the ambient fluid, H^*, was equal to r^* in the circular tube, while in the rectangular one it was $H^* = 11$ cm. For the experiments in the circular cross-section channel the discharge was $Q^* = 65$, 144 ml/s, for the experiments in the rectangular cross-section channel the discharge was $Q^* = 80$, 144 ml/s. In the lock-release tests the height of the intruding current in the lock was $h_0 = 3.2$, 4.0, and 5.0 cm.

We tried to cover almost uniformly the range $S = 0 \div 1$, to analyze the behavior of gravity currents within a huge variety of linear stratification, performing more than 70 different experiments.

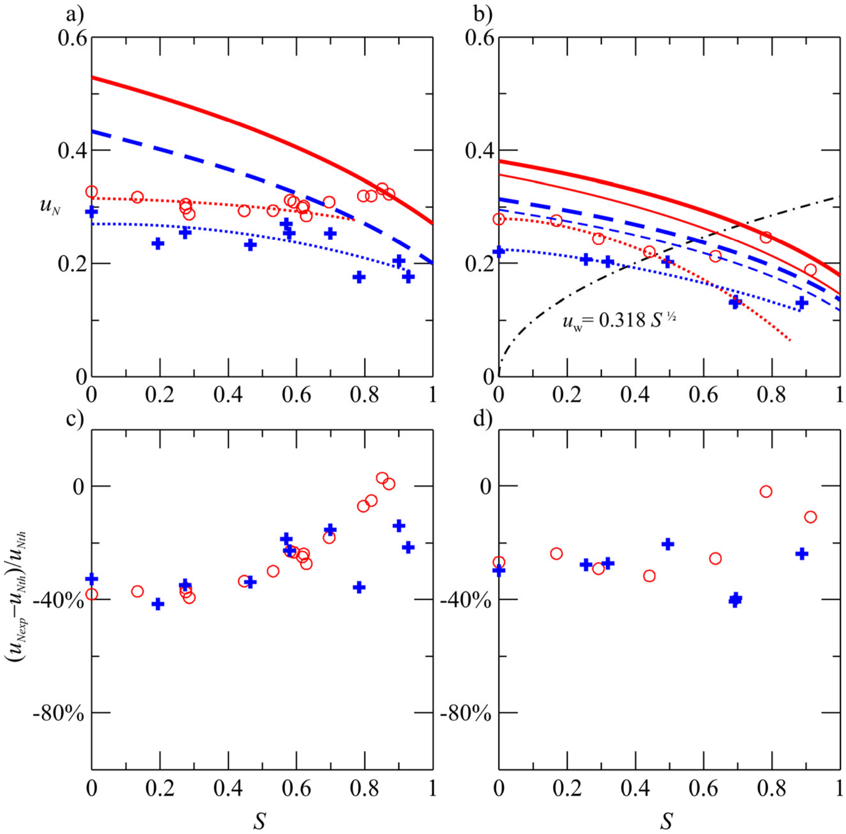

Figure 2. Experimental and theoretical values of non-dimensional u_N for constant influx experiments. (a) and (b) show the results for circular and rectangular cross-section channel, respectively. (c) and (d) show the relative error. $Q^* = 144$ ml/s (solid line for theory, circles for the experiments) in panel a) and b), $Q^* = 65$ ml/s (dashed line for theory, crosses for the experiments) (panel a) or $Q^* = 80$ ml/s (panel b). The dotted lines interpolate with a parabola the measured values. The thin lines in panel b) refer to theoretical values by using Fr as reported in *Huppert & Simpson* (1980). The dash-dot line in panel b) represents the first mode or the internal wave celerity u_w, theoretically known for rectangular cross-section (see, e.g., *Ungarish* 2009).

4 RESULTS

In Fig.2(a) the front speed of the current is shown compared with the theoretical model, for the circular cross-section for two values of the discharge and for different values of S. The density of the intruding current ρ_c and the value of H^* are kept constant and equal to 1100 kg/m^3 and 9.5 cm, respectively. Fig. 2(b) shows similar results but for the rectangular cross-section. In this last set of experiments the comparison is also made by considering the experimental Froude number illustrated in *Huppert & Simpson* (1980):

$$Fr(h_N) = \begin{cases} 1.19 & \left(0 \le \dfrac{h_N}{H^*} \le 0.075\right), \\ \dfrac{1}{2}\left(\dfrac{h_N}{H^*}\right)^{-1/3} & \left(0.075 \le \dfrac{h_N}{H^*} \le 1\right). \end{cases} \tag{5}$$

The results show that for circular cross-section the theoretical model overestimates the speed of propagation. This is coherent with past results in similar condition, but the overestimation for limited values of S is quite large. The value of Q^* also seems to play a role, since larger discharge means a better agreement for $S \rightarrow 1$. For rectangular cross-section a similar overestimation is evident but with slightly lower values with respect to the circular cross-section. If the comparison is made with the experimental value of the Froude number by Huppert & Simpson, the discrepancy reduces but it is still relevant, in particular for $S \rightarrow 0$. We believe that the internal waves (the first mode celerity is drawn in Fig.2b) play a role in the dynamics of the intruding current.

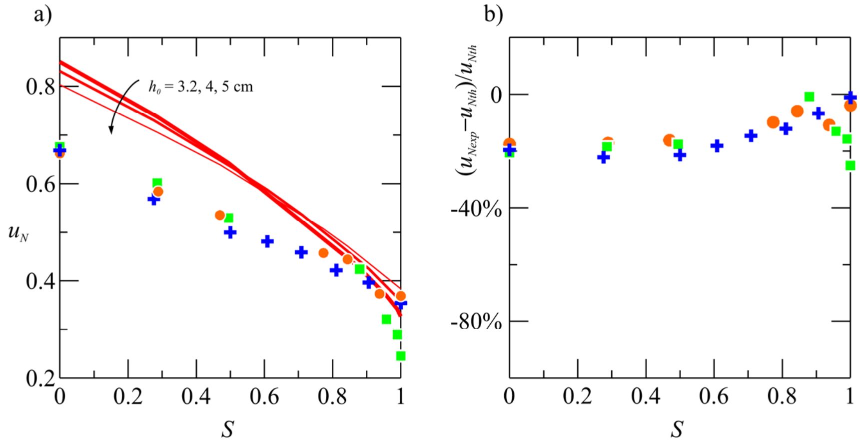

Figure 3. Experimental and theoretical values of non-dimensional u_N for lock release experiments for circular cross-section channel. (a) shows the values for h_0= 3.2 cm (thick line for theory, squares for experiments), h_0= 4.0 cm (mid thickness line for theory, crosses for experiments), and h_0= 5.0 cm (thin line for theory and circles for experiments). Panel (b) shows the relative error.

Fig.3(a-b) shows the front speed of the current and the relative discrepancy theory-experiments for the three sets of experiments with lock release in the circular cross-section channel. The density of the current and the height of the ambient fluid are again equal to 1100 kg/m^3 and 9.5 cm, respectively. The three sets differ for the height of the intruding current in the lock, equal to 3.2, 4.0 and 5.0 cm, respectively. A coherent trend is observed with minimal differences for the three values of h_0. As for the constant influx experiments, the error decreases for increasing S. However for $S \rightarrow 1$ a fast drop of agreement is evident for h_0 = 3.2 cm. The general agreement is better than for the experiments with constant influx, with a maximum discrepancy smaller than 20%.

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