BOTTOM STRESS IN NON STATIONARY FREE SURFACE FLOW

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Abstract: This paper describes studies of the bed friction factor in non-stationary free surface flow, for the specific case of bores moving in shallow water and breaking waves on beaches. Data from a set of experiments carried out in a laboratory flume, including measurements of water level displacement and fluid velocity through LDV, are used to evaluate the bottom stress and the mean flow velocity. The results of other measurement techniques in similar conditions are used to critically assess our results. It is found that the friction factor is higher and the transition from laminar to turbulent boundary layer takes place at higher Re numbers than values obtained in separate studies of turbulent bottom boundary layer including an additional source of turbulence.

INTRODUCTION

Bottom friction and boundary layer dynamics play a major role in many non-stationary free surface physical flows such as roll waves and bores in shallow water and over beaches. This subject is of interest because the stability analysis of free surface flows aiming to detect the presence of roll waves is sensitive to the friction, and also, bore dynamics in shallow water and the maximum run-up of waves over beaches depends on the bottom friction.

Flow resistance is usually expressed through a friction factor. The friction factor is an integral expression of the efficiency in fluid momentum transport and is useful in modeling stream processes without resolving the detail in the turbulence and the velocity distribution. In general, in stationary flows the resisting force per unit length acting on a control volume is the product of the wetted perimeter of the section and the mean intensity of the boundary stress. If the flow is accelerating there are forces working

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conservatively and other forces working dissipatively. In order to derive the actual resistance, only the latter need to be considered. According to Rouse (1965), the resisting force F can be expressed as a:

$$F = f\left(h, U, k, \rho, \nu, g, \xi, \chi, \eta, \partial h/\partial t\right) \tag{1}$$

F is a function of the mean local water depth h, the mean velocity U, a scale length of the roughness k, the fluid density ρ, the fluid viscosity ν, the acceleration of gravity g and of several dimensionless parameters describing the shape of the cross section, the bottom profile and the channel plan, the rate of change of depth with time $\partial h/\partial t$. Using Buckingham theorem results in:

$$\frac{F}{\rho U^2 h} = f\left(\frac{k}{h}, \frac{Uh}{\nu}, \xi, \chi, \eta, \frac{U}{\sqrt{gh}}, \frac{\partial h/\partial t}{U}\right) \tag{2}$$

The dependence on the first five groups on the right hand (relative roughness and Reynolds number and shape parameters) is well studied. The dependence on the Froude number has been assessed especially in roll waves studies (Rouse, 1965, Brock, 1966).

The dependence on the unsteady parameter $\left(\partial h/\partial t\right)/U$ has been less studied. Bottom friction is the cumulative effect of perturbations in the bed on the adjacent flow and the corresponding generation of a boundary layer. The characteristics of the boundary layer depends on the nature and intensity of these perturbations. Smits and Wood (1985) introduce the definition of 'simple' flow if the perturbation is weak and self preservation still acts (i.e. the boundary layer can be described using a steady state approximation but with local, changing variables). A severe perturbation can either maintain the boundary layer, or destroy it.

The possible perturbing mechanisms include an extra strain rate, a change in wall conditions (e.g. in roughness), a pressure gradient.

Extra strain can be generated by longitudinal streamline curvature, streamline convergence and divergence, compression and dilation. Compression and dilation in breaking waves and bores can be a consequence of air bubbles strongly mixed in the fluid flow. The effects of an extra strain rate is more evident far away from the bottom, where the basic strain rate is smaller.

A step change in wall conditions and a rapid change in pressure gradient are the most effective in altering the bottom stress. In this case we consider a uniform bottom, and neglect the former and focus on the latter.

A pressure gradient accompanies a distortion in the free surface during the wave breaking process, hydraulic jumps and bores. It can also be generated by streamline curvature do to bottom curvature. In the swash zone, during wave uprush the pressure gradient is favorable and tends to stabilize the turbulent boundary layer and can revert it to a laminar-like state (see Warnack and Fernholz, 1998). During downrush the pressure gradient is adverse and can generate a bore with possible separation and zero bottom stress.

There are two other very important perturbations that can strongly modify the boundary layer structure, i.e. infiltration/exfiltration and externally generated turbulence. Infiltration is possible in porous bottom and acts in stabilizing the boundary layer and increasing bottom stress, whereas exfiltration increases the boundary layer thickness and reduces the stress (see Conley and Inman, 1994). These phenomena are also important for sediment transport (Butt et al., 2001). Externally generated turbulence acting on an oscillatory boundary layer increases the bottom stress (Kozakiewicz et al., 1998).

There are several methods and techniques adopted for evaluating the friction factor. In steady state free surface flow it is straightforward and very precise to measure the free surface inclination and the fluid velocity; the use of 2-D LDA, or LDA in a specific arrangement a as Laser Gradient Meter (Obi et al., 1996), and hot film probes also allow the measurement of turbulence fluctuations near the bottom, by extrapolating the stress at the bed; some specific Pitot Tubes have also been used (e.g. Preston's method, Preston, 1954).

In the experiments described herein, turbulence measurements were made using LDV at approximately 0.5 mm above the bottom in order to investigate the bottom stress under breaking waves on a beach.

The purpose of this study is to evaluate the friction factor in a typical unsteady free surface flow, such as uprush-downrush in the swash zone on an impermeable bottom.

EXPERIMENTAL APPARATUS AND TESTS

Experiments were carried out in a wave flume 48 m long, 0.8 m wide, and 0.8 m depth (Fig. 1). Three concrete bottom sloping beaches, 1:5, 1:10, and 1:15, were constructed opposite to the wave maker with their surfaces finished to reduce the roughness. The geometric scale of roughness was evaluated, for each bottom configuration, through a detailed analysis of specimens using a Laser interferometric pick-up transverse unit. The average height value of the crests was around 30 μm for all bottoms.

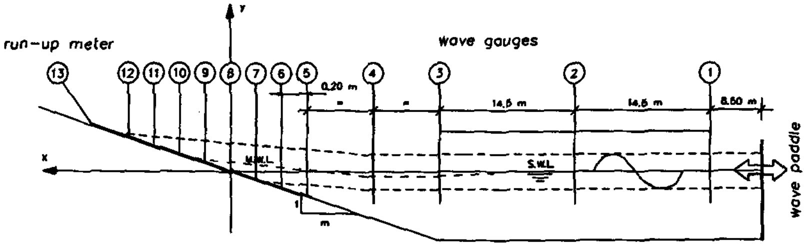

Fig. 1. Experimental set-up and location of wave gauges

Three regular periodic waves of 2 s, 2.5 s and 3 s were generated in the flume with a still water level in front of the paddle equal to 40 cm. The complete set of regular wave tests performed is summarized in Table 1; subscripts 0 refer to data estimated in deep water using linear wave theory.

Table 1. Experimental wave conditions

Test	H_0, cm	H_0, cm	T, s	H_0/L_0
RH040T20	3.6	3.5	2.0	0.006
RH040T25	3.3	3.4	2.5	0.003
RH040T30	3.3	3.8	3.0	0.002

Instantaneous velocities in the swash zone were measured by a Laser Doppler Velocimeter (LDV) system in forward scatter, at several equally spaced points with 1 mm step, starting at 0.5 mm from the bottom, measured along the vertical at three different sections. The first section was at the intersection of the still water level and the slope (sec. 8), the second +20 cm shoreward (sec. 9), and the last at -20 cm seaward (sec. 7). Many runs for each regular wave were necessary to cover the whole vertical measure for each section, and a repetitiveness analysis was succesfully carried out. This was done by checking the wave height and wave period collected by gauges 1-3 for the whole set of tests.

For each combination of regular wave condition and bottom slope, the breaking type was estimated by the surf similarity parameter ξ_b (Table 2). The experiments cover the range of plunging plus bore ($0.4<\xi_b<0.8$) plunging ($0.8<\xi_b<1.14$), collapsing ($1.14<\xi_b<2.0$) and surging breakers ($\xi_b>2.0$).

Table 2. Breaking range covered with all tests

Bottom slope	1:5	1:10	1:15
T, s	ξ_b^*	ξ_b^*	ξ_b^*
2.0	1.83	0.92	0.61
2.5	2.35	1.17	0.78
3.0	2.72	1.36	0.91

*Surf similarity parameters ξ_b estimated by the relation $\xi_b=1.45\tan\theta/(H_0/L_0)^{0.36}$, (Gourlay, 1992).

The acquisition time of each test was 300 s and the acquisition rate was 100 Hz. More details on the experimental apparatus and tests can be found in Petti and Longo (2001).

BOTTOM STRESS AND FRICTION FACTOR

The friction factor f_w for periodic flows is usually defined as (Jonsson and Carlsen, 1976):

$$f_w = \frac{2\tau_{max}}{\rho U_{0max}^2} \tag{3}$$

where τ_{max} is the maximum bottom stress, U_{0max} the amplitude of the outer flow field velocity and ρ the water density.

In the present experiments the turbulence at the first level (~0.5 mm over the bottom) in the longitudinal direction was used to evaluate the bottom stress. The method is based on the following considerations.

Velocity fluctuations near the wall are scaled with friction velocity. Several experiments in open channel flows with smooth walls using hot-films (Nakagawa et al., 1975) and LDV (Nezu and Rodi, 1986) suggest that a fitting function in the intermediate region ($0.1<y/h<0.6$) is:

$$\frac{u'}{U_*} = 2.30 \exp(-y/h) \tag{4}$$

where u' is the r.m.s. of the turbulent velocity component along the stream, U_* the friction velocity, y is the distance from the bottom and h is the water depth. Similar distributions, i.e. exponential decay, have been described for the two others components v', w' of the fluctuating velocity and for turbulent energy κ. These distributions have been shown to be independent of the main flow condition, regardless of the Reynolds and Froude number.

In the wall region ($y/h\leq 0.15$), for very large Reynolds number $\mathrm{Re}_* = U_* h/\nu$, Monin and Yaglom (1971) proposed the asymptotic relationship :

$$\frac{u'}{U_*} \rightarrow 2.3. \tag{5}$$

The available experiments on turbulence near a rough bottom (Grass, 1971, Nezu, 1977) suggest that the effects of roughness are a reduction of the asymptotic value in Eq.(5) for increasing roughness, without modification of the decaying function shape.

In the present experiments the instantaneous water depth h can be zero and thus the assumption of the maximum local water level δ as a vertical length scale is more suitable. For the whole set of experiments and all sections, the measured value of δ was between 10 and 70 mm, with a non dimensional value y/δ at the first level of measurement (y=0.5mm) less than 0.05, thus the approximation given in Eq.(5) is substantially valid. In the following, Eq.(5) will be used to estimate the maximum friction velocity U_{*max} using the measured fluctuating values u'.

We assume for the friction factor the standard definition given in Eq.(3), neglecting the time varying structure of the bottom stress. We also define the Reynolds number for a purely oscillatory flow:

$$\mathrm{Re} = \frac{aU_{0max}}{\nu} \tag{6}$$

where a is the free stream amplitude.

Unfortunately, LDV measurements experience many drop off during the wave period, due to bubble presence or signal unlock, and cannot be used to determine the outer velocity with confidence. A more reliable depth averaged velocity (here assumed equal to the outer velocity) can be computed using the mass balance equation:

$$U = \frac{1}{h}\frac{dV}{dt} \tag{7}$$

with h the instantaneous stream thickness in the section of interest and V the volume of water stored shoreward beyond the section. The volume is computed using the instantaneous free surface levels measured by the probes assuming a linear profile between adjacent probes. The velocity (not reported) has a saw-tooth profile similar to that which occurs in the bore-like broken waves (Schäffer and Svendsen, 1986).

In Table 3 the estimated velocity U_{0max} and maximum friction velocity U_{*max} are reported for each test.

Table 3. Outer maximum velocity and maximum friction velocity in present study.

Bottom slope	1:5		1:10		1:15	
T, s	U_{0max}, m/s	U_{*max}, cm/s	U_{0max}, m/s	U_{*max}, cm/s	U_{0max}, m/s	U_{*max}, cm/s
Section 7						
2.0	0.94	15.4	0.55	6.4	0.34	8.6
2.5	0.85	12.1	0.62	5.2	0.58	15.0
3.0	0.72	16.3	0.87	6.4	0.68	6.7
Section 8						
2.0	0.81	27.4	0.41	5.0	0.26	4.7
2.5	0.74	2.4	0.43	5.2	0.57	5.8
3.0	0.66	8.7	0.82	5.3	0.51	5.7
Section 9						
2.0	0.46	5.8	0.32	4.9	1.00	7.2
2.5	0.47	3.6	0.41	2.8	1.13	13.2
3.0	0.64	3.0	0.82	4.4	0.44	16.5

Once the maximum friction velocity U_{*max} is known the bottom stress $\tau_{max}=\rho U_{*max}^2$ and the friction factor can be easily computed.

To estimate the Reynolds number we need the free stream amplitude a, that for a saw-tooth shaped outer velocity, independent on the acceleration and deceleration value, is given by:

$$a = \frac{U_{0max}T}{8} \tag{8}$$

Comparison is made with experiments reported in Jensen et al. (1989) (hereafter JSF) and in Kozakiewicz et al. (1998) (hereafter KSFD). JSF's experiments refer to a turbulent oscillatory boundary layer at high Reynolds numbers, KSFD's experiments refer to oscillatory boundary layer in a pulsating tunnel with an extra source of turbulence due to a grid in the upper part of the tunnel.

The friction factor versus Reynolds number for the present experiments is plotted in Fig.3, and compared with similar results obtained by JSF and KSFD. KSFD found that the outer flow turbulence generated by the grid, penetrates into the bottom oscillatory boundary layer and increases the friction factor, also inducing an earlier transition to turbulence. In particular, we found that the transition takes place when the Reynolds number has a value of about 7×10^4 rather than 1.6×10^5 found by JSF (also in the absence of an extra source of turbulence).

In the present swash zone measurements we find different behaviors for different wave breaking type. For plunging and collapsing there is no a clear transition from laminar to turbulent flow. It appears that, for relatively small Re numbers (less than 10^5) the boundary layer is laminar during up-rush for only a short time, and becomes turbulent as soon as the jet flow of the breaker reaches the bottom with high vorticity and turbulence. Under these conditions the friction factor decreases with Re and tends to collapse onto the laminar curve, as relaminarization would occur. For surging and plunging breakers plus bore, however, the friction factor behaves as in the experiments of KSFD and JSF, with a clear transition.

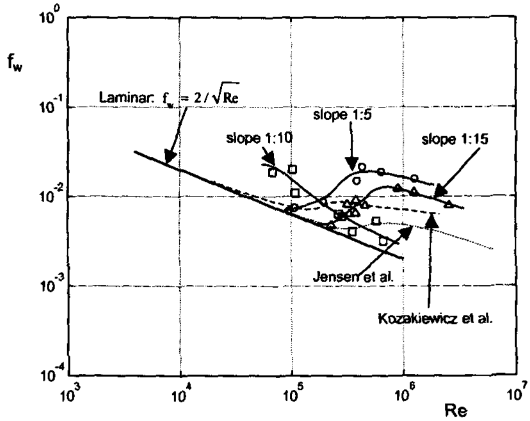

Fig. 3. Friction coefficient versus Re for the present study (symbols). Dot line: Jensen et al. (1989) turbulent oscillatory flow; Dashed line: Kozakiewicz et al.(1998), turbulent oscillatory flow plus external generated turbulence.

The transition takes place at Reynolds numbers between approximately 10^5 and 2×10^5 for surging and for plunging (plus bore) breaking waves, which is comparable to values obtained in an oscillatory turbulent boundary layer by JSF, though higher than those obtained by KSFD, due to a stabilizing effect. This stabilizing effect is possibly the result of the strong favorable pressure gradient in these tests. The computed friction factors are higher than those obtained by KSFD (which included the presence of an additional source of turbulence).

The phase lead of the bottom stress with respect to the outer flow, for a laminar oscillatory boundary layer is equal to 45°, and tends to reduce at high Reynolds number in the turbulent regime. In our tests the measured phase has also decreasing values with increasing Re, which is typical of the transition zone. The results are shown in Fig.4.

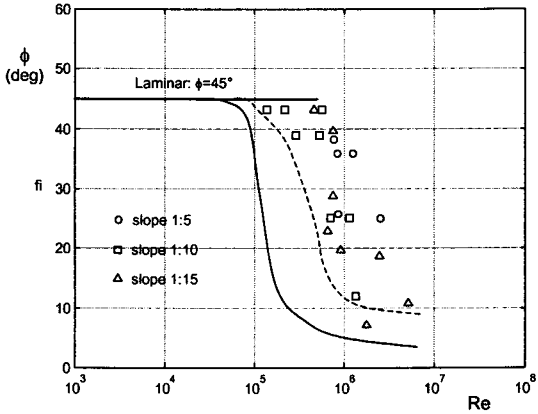

Fig. 4. Phase lead of the bottom stress over the outer flow velocity. Thick line: fitting curve for boundary layer with external generated turbulence (Kozakiewitz et al., 1998). Dashed line: fitting curve for simple oscillatory boundary layer (Jensen et al., 1989). Symbols: values measured in the present experiments.

In order to evaluate the time varying bottom stress, the momentum balance equation is integrated. The results are slightly sensitive to the bottom roughness, and in this case the measured geometrical value was used.

The momentum balance equation in the main stream direction is (Fredsøe and Deigaard, 1992):

$$\rho \frac{\partial (U-U_0)}{\partial t} + \frac{\partial \tau}{\partial y} = 0 \tag{9}$$

where the boundary layer approximation has been used. Assuming a logarithmic velocity profile in the turbulent boundary layer:

$$\frac{U}{U_*} = \frac{1}{k} \ln\left(\frac{30y}{k_N}\right) \tag{10}$$

(y_o is the reference level, equal to $k_N/30$ according to Nikuradse; k_N is the bed roughness) the momentum balance equation can be integrated from the bottom to the upper limit of the boundary layer, where the shear stress is assumed to be zero:

$$\rho \int_{y_o}^{y_o+\delta} \frac{\partial (U-U_0)}{\partial t} dy = \tau_b \tag{11}$$

assuming $U=U_0$ at the upper limit of the boundary layer and $\tau_b = \rho U_*^2$.

In Fig. 5 the computed friction velocity and boundary layer thickness is reported for a test case, assuming a bottom roughness equal to the geometric measured bottom roughness. The maximum friction velocity is higher for fast accelerating flow during uprush. The mean bottom stress is slightly shoreward.

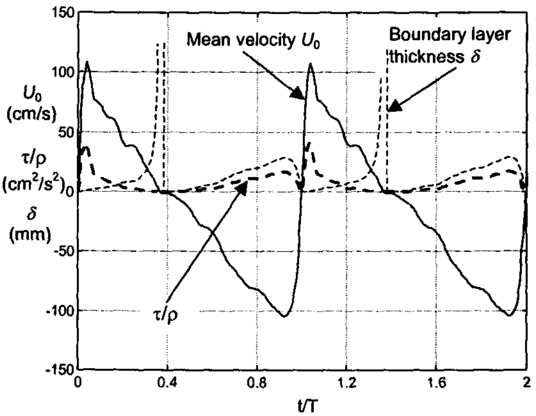

Fig. 5. Computed bottom stress and boundary layer thickness in the lower section using the integral momentum method; k_N=50x10^{-6} m, T=3.0 s, section 8, 1:10 bottom slope.

CONCLUSIONS

The friction factor in a typical non stationary free surface flow generated by a wave breaking depends on the type of breaker. Using lab measurements the calculation of the friction factor gave similar results to those obtained with other lab measurements of a turbulent oscillatory boundary layer (Jensen et al., 1989) and an oscillatory boundary layer with extra source of turbulent energy (Kozakiewicz et al., 1998), with a clear transition from laminar to turbulent flow

Friction factor is higher than those obtained in these two other studies above .

The transition to turbulence agrees with the findings of Jensen et al. (1989) at a Re of approximately 10^5 but differs from that obtained by Kozakiewicz et al. (1998), considered to be due to the different physical characteristics of the two experiments.

For plunging and collapsing breakers transition (from laminar to turbulent flow) probably occurs at lower Re due to the breaking jet. A relaminarization appears at high Reynolds number.

The phase lead of the bed shear stress over the outer (mean) flow velocity is decreasing for increasing Re number, as happens in transition flows.

The average bottom stress, computed using the integrated momentum equation is slightly positive.

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