AGU Fall Meeting - *San Francisco, 14-18 December 2015*

H42A-01 COMPLEXITIES OF FLOW AND TRANSPORT IN POROUS MEDIA ACROSS DIVERSE DISCIPLINES III

Gravity-driven flow of non-Newtonian fluids in heterogeneous porous media: a theoretical and experimental analysis

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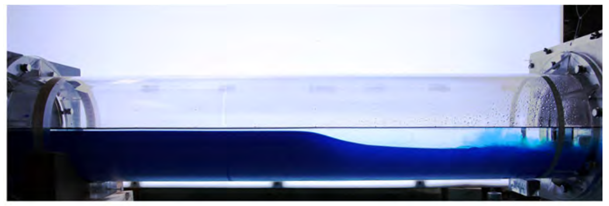
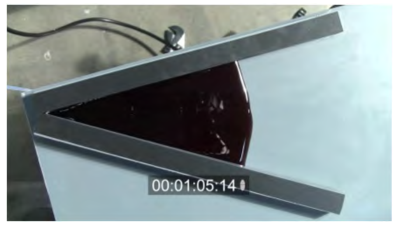
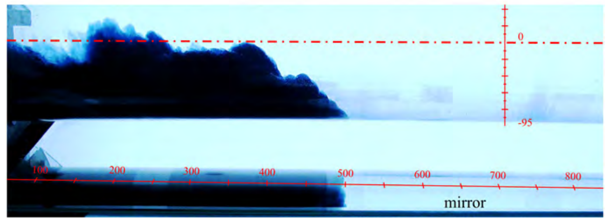
Motivation (I)

"Gravity current" (GC): one fluid moves in horizontal direction into another fluid because the densities are different.
The pressure/buoyancy driving is "balanced" by either inertial, or viscous, adjustment of the fluid.

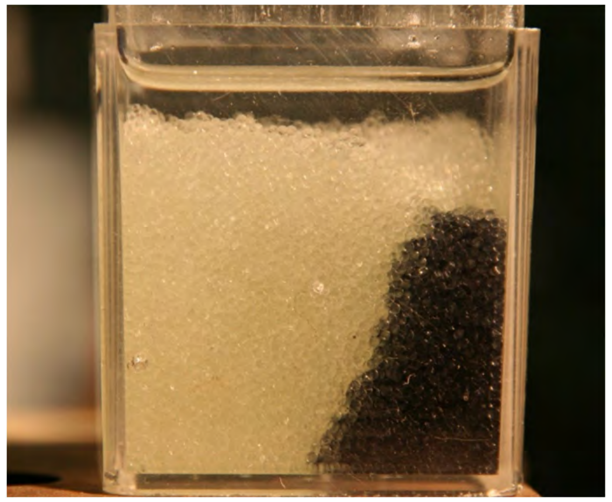






Viscous GC in porous media (PM) occur in many environmental processes:

- enhanced oil and heat recovery
- groundwater remediation
- carbon dioxide sequestration
- saltwater intrusion



Motivation (II)

In PM flow, fluid rheology is often non-Newtonian (NN):

- polymeric suspensions in enhanced oil recovery
- pollutants in environmental modelling
- muds in well drilling
- crude oil in reservoir engineering
- fluid carriers for nanoparticles in soil remediation
- injectable biomaterials in biological applications (orthopaedics, dental)

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Need of efficient tools for understanding and prediction of NN gravity currents in PM

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simplified (analytical) models provide reliable guidelines

↓

experimental verification

Background and objective

- Analytical and numerical solutions (with experimental verification) are available in the literature for Newtonian GC in homogeneous PM (Huppert 2006, Ungarish 2009); reference cases: plane (Huppert and Woods 1995) and radial (Lyle et al. 2005) geometry
- Solutions for thin currents ($H/L \ll 1$) are self-similar (intermediate asymptotics)
- Solutions for flow of non-Newtonian power-law fluids were recently derived (Di Federico et al., *JNNFM* 2012a,b) and experimentally verified (Longo et al., *JFM* 2013) for plane/radial case
- Natural PM are inherently heterogeneous. Spatial heterogeneity affects GC spreading rate and extent and shape of porous domain invaded/reached

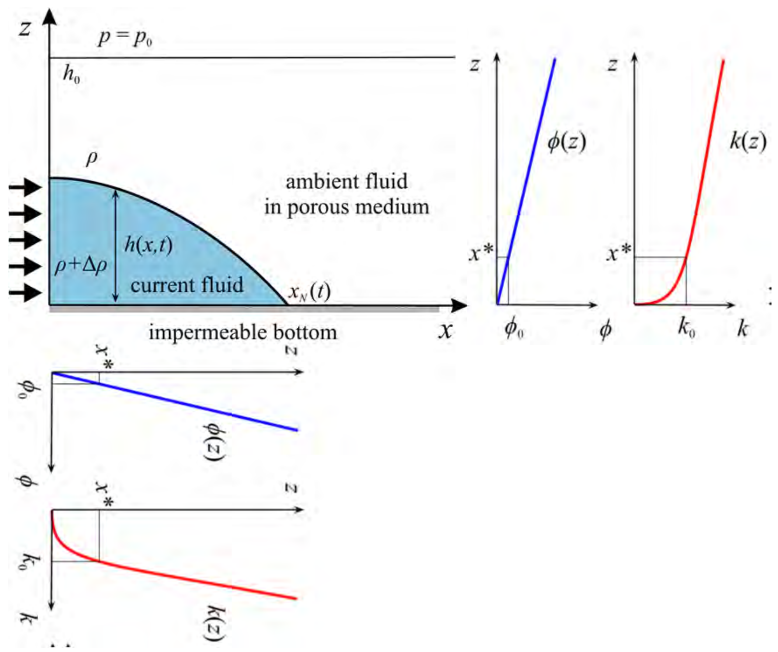
⇩

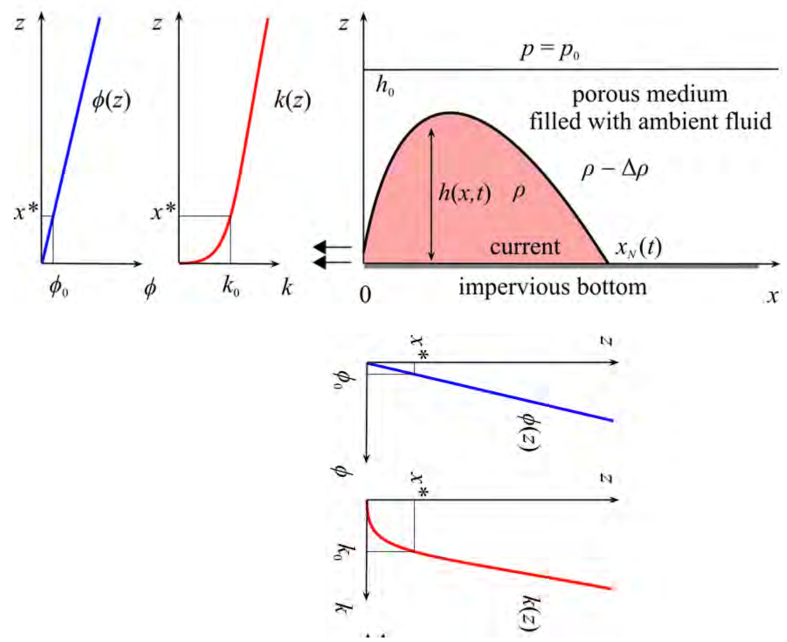
Need to explore the combined effect of rheology and heterogeneity considering non-Newtonian flow in media with variable properties

Scenarios

- Geometry: 2-D plane flow
- Fluid rheology: power-law fluid
- Spatial heterogeneity: permeability/porosity varying transverse or parallel to flow direction
- Type of release: instantaneous injection of constant mass, continuous injection, instantaneous injection of mound draining freely out of formation (dipole flow)

$$V = qt^{\alpha}$$

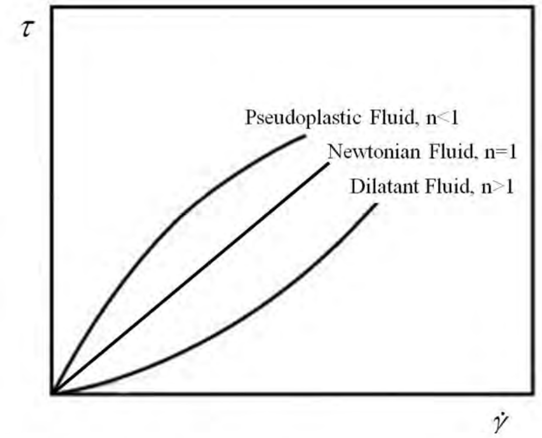




Flow law in PM

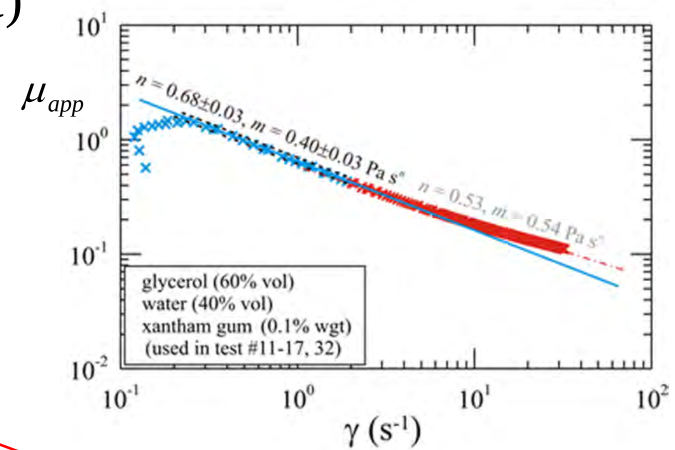
Stress-shear rate relationship, Ostwald–deWaele fluid, simple shear

$$\tau = \tilde{\mu}\left|\dot{\gamma}\right|^{n-1}\dot{\gamma} = \mu_{app}\dot{\gamma}$$



$\tilde{\mu}$ consistency index $[ML^{-1}T^{n-2}]$

n flow behavior index
- $n < 1$ shear-thinning (pseudoplastic)
- $n = 1$ Newtonian ($\mu_{app} \to \mu$)
- $n > 1$ shear-thickening (dilatant)



Eq. of motion for power-law fluids in porous media (e.g. Shenoy, 1995)

p pressure
$\mathbf{v}$ seepage flux
k permeability $[L^2]$
ϕ porosity
$C_t = C_t(n)$ tortuosity

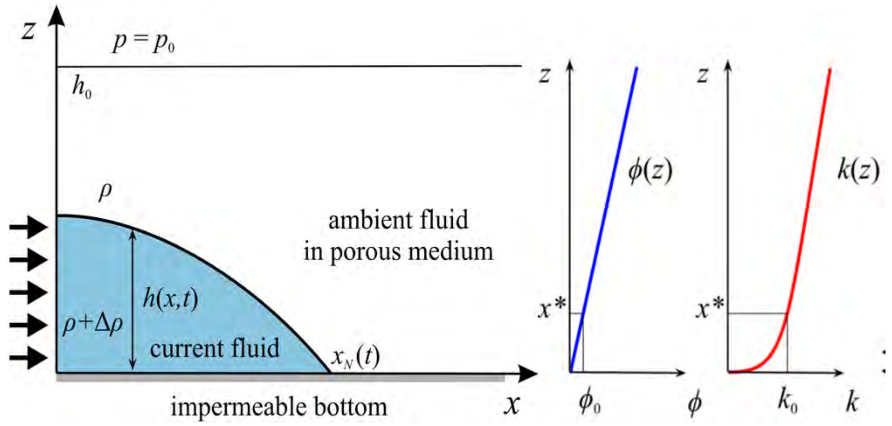
$$\nabla(p + \rho g z) = -\frac{1}{\Lambda k^{(1+n)/2}}\left|\mathbf{v}\right|^{n-1}\mathbf{v},$$

$$\Lambda = \Lambda(\phi, \tilde{\mu}, n) = \frac{1}{2C_t}\left(\frac{50}{3}\right)^{(n+1)/2}\left(\frac{n}{3n+1}\right)^{n}\frac{\phi^{(n-1)/2}}{\tilde{\mu}}$$

Problem formulation

i) immiscible displacement; ii) thin intruding current; iii) no motion of ambient fluid
iv) no capillary effects; v) instantaneous/continuous injection

$$V = qt^{\alpha}$$



Permeability/porosity spatial variation

$$k(z) = k_0 (z/x_0)^{\omega-1}$$

$$\phi(z) = \phi_0 (z/x_0)^{\gamma-1}$$

Flow law $$u_x = (\Lambda \Delta\rho g)^{1/n} k^{(1+n)/2n} \left(-\frac{\partial h}{\partial x} \right)^{1/n}$$

Mass balance $$\frac{\partial}{\partial x} \int_0^h u\,dz + \frac{\partial}{\partial t} \int_0^h \phi\,dz = 0$$

Condition at the current tip $$h(x_N(t), t) = 0$$

Global mass balance $$\int_0^{x_N(t)} \int_0^h \phi\,dz\,dx = qt^{\alpha}$$

α=0 constant volume
α=1 constant inflow rate

Self-similar solution

Dimensionless coordinates

PDE $\eta = F_1^{F_4} x / t^{F_2}$ Similarity Variable $\eta_N = \eta(x_N)$ $\zeta = \eta/\eta_N \in [0,1]$

$x_N(t) = \left(\eta_N / F_1^{F_4}\right) t^{F_2}$ Current tip position $F_i = F_i(\alpha, n, \gamma, \omega), i = 1,\dots,5$ Numerical factors

$h(x,t) = F_1^{F_4/\gamma} \eta_N^{F_5/\gamma} t^{F_3} \Phi(\zeta)$ Current height $\Phi(\zeta)$ Shape function

ODE

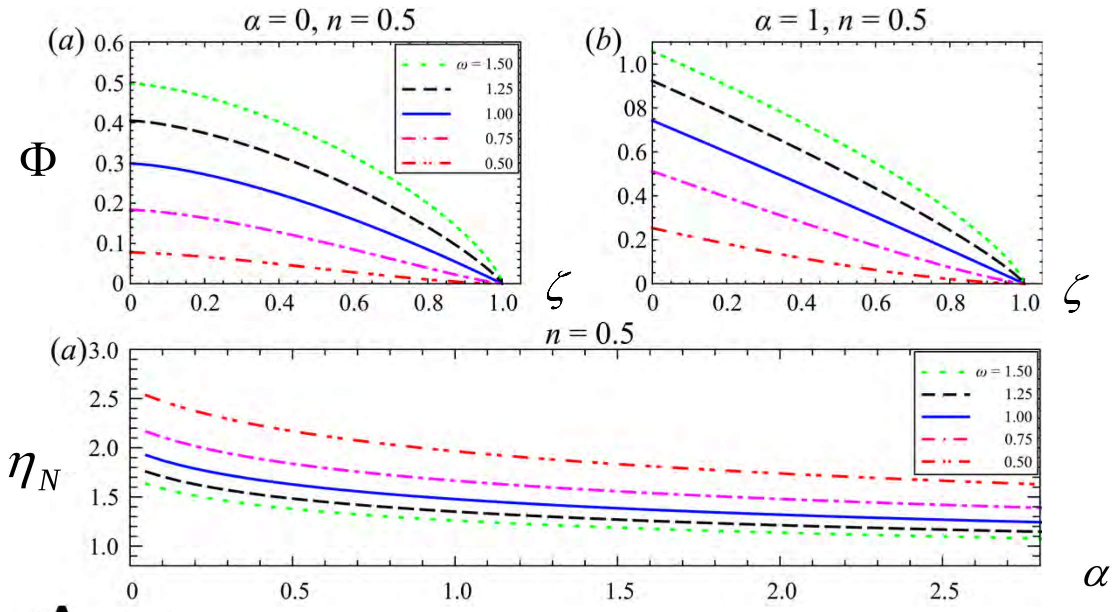
$$\frac{d}{d\zeta}\left[\Phi^{F_1}\left(-\frac{d\Phi}{d\zeta}\right)^{1/n}\right] + F_2 \zeta \Phi^{\gamma-1} \frac{d\Phi}{d\zeta} - F_3 \Phi^{\gamma} = 0 \qquad \eta_N = \left(\int_0^1 \Phi^{\gamma} d\zeta\right)^{-1/(F_5+1)} \qquad \Psi(1) = 0$$

Closed-form solution for α=0, numerical for α>0

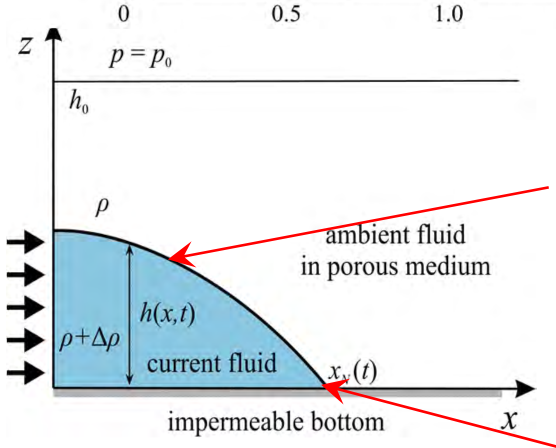
$$\Psi(\zeta) = \left(\frac{F_{20}^n}{F_5 \gamma^{n-1}}\right)^{F_5/[\gamma(n+1)]} \left(1-\zeta^{n+1}\right)^{F_5/[\gamma(n+1)]}; \eta_N = \left[\frac{1}{\gamma}\left(\frac{F_{20}^n}{F_5 \gamma^{n-1}}\right)^{F_5/[\gamma(n+1)]} \frac{F_5}{(F_5+1)(n+1)} B\left(\frac{1}{n+1}, \frac{F_5}{n+1}\right)\right]^{-1/F_5+1}$$

$$F_{20} = F_2(\alpha = 0)$$

Results



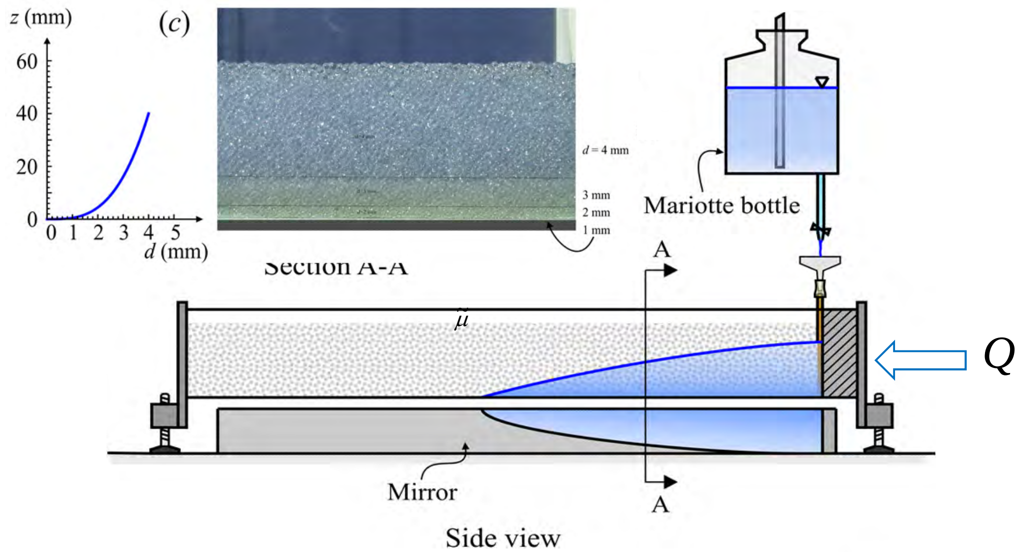
$\gamma = 1$



Current profile $h(x,t) \propto t^{F_3}\Phi(\zeta)$

Propagation distance $x_N(t) \propto t^{F_2}\eta_N$

Experimental setup (I)

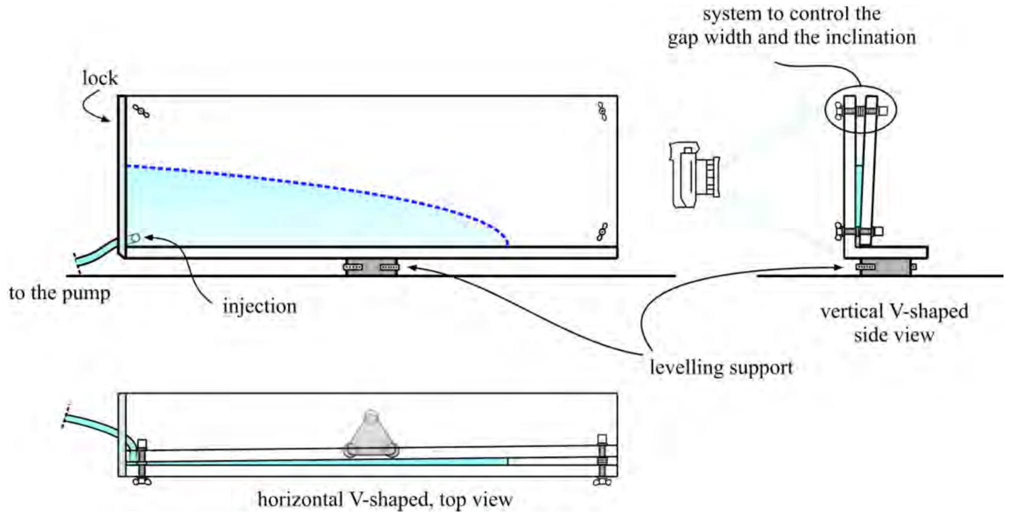


Direct simulation

- Porous medium=glass ballotini
- Vertical heterogeneity (k) reproduced via different strata with different diameter and thickness

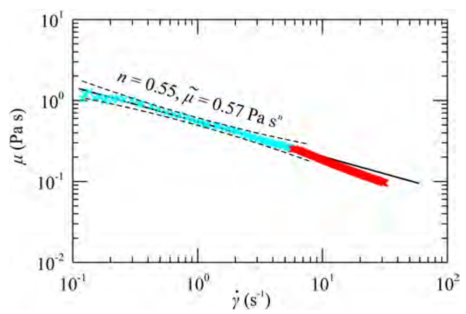
Analogue simulation

- Hele-Shaw analogy formally developed for NN flow
- Vertical/horizontal heterogeneity (k and ϕ) reproduced via cell with variable aperture in horizontal/vertical direction



Experimental setup (II)

- Different power-law fluids (mix Glycerol, water and Xanthan gum)
- Fluid rheological parameters determined a priori via rheometer
- Two high-resolution photocameras employed to record current tip and profile
- Constant inflow rate Q ($\alpha = 1$)
- Horizontal (A) and vertical (B) variation of properties

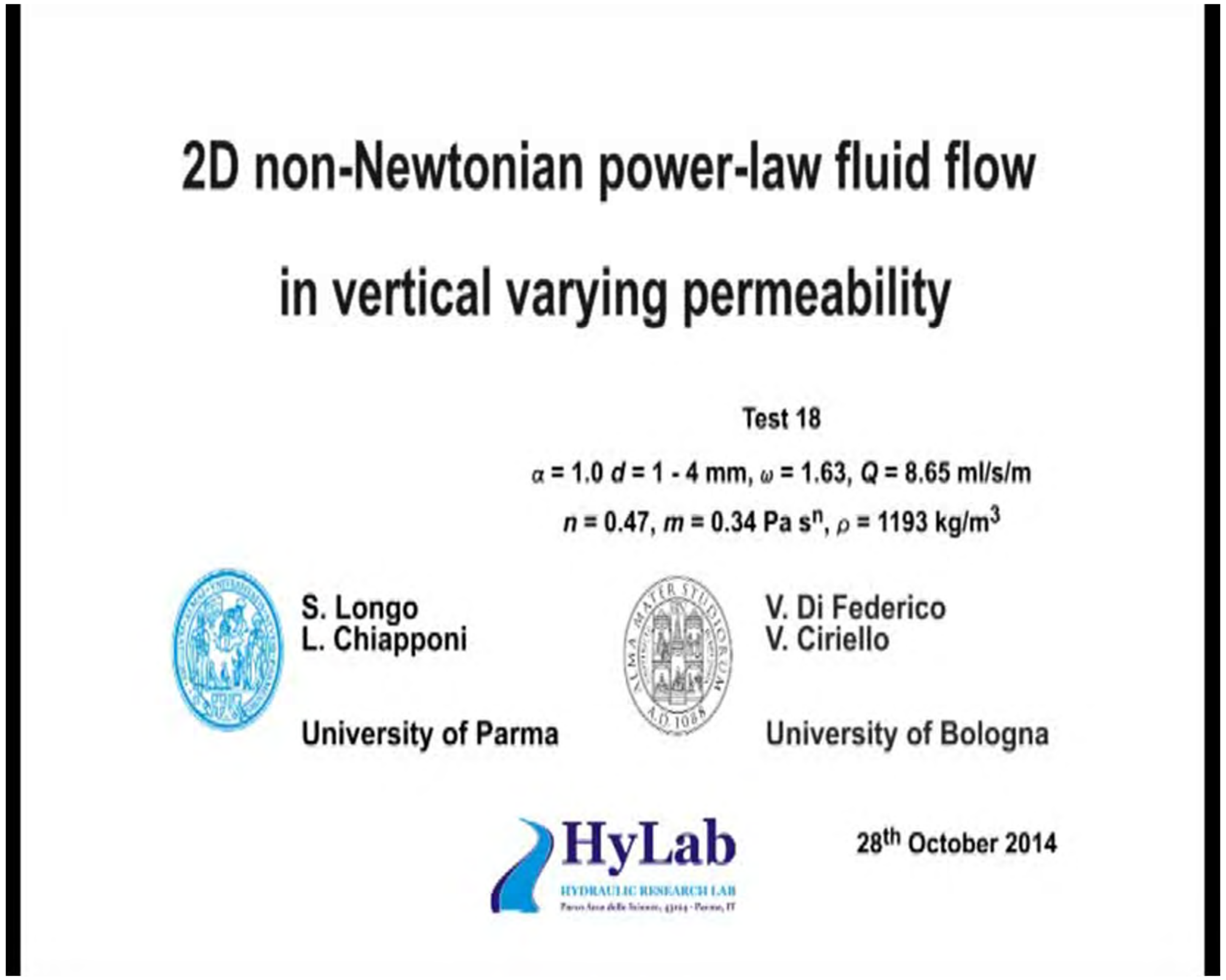


A

#	β	δ	b_0	$\tilde{\mu}$ (Pa s^n)	n	ρ (kg m^{-3})	q (ml s^{-1})
6	3	1	0.00205	0.34	0.47	1193	0.609
7[a]	3	1	0.00308	0.34	0.47	1193	0.600
8	3	1	0.00308	0.34	0.47	1193	0.317
10	3	1	0.00308	0.20	0.45	1100	0.580
11	3	1	0.00308	0.20	0.45	1100	1.59
12	3	1	0.00304	0.20	0.45	1100	1.37

B

#	ω	γ	b_0	μ (Pa s^n)	n	ρ (kg m^{-3})	q (ml s^{-1})
1	4	2	0.00469	0.10	1	1230	0.639
2	4	2	0.00469	0.34	0.47	1193	0.296
3	4	2	0.00469	0.34	0.47	1193	0.233
13	4	2	0.00469	0.20	0.45	1100	2.05
14	4	2	0.00469	0.20	0.45	1100	2.28
15	4	2	0.00469	0.20	0.45	1100	0.27
19	4	2	0.09800	0.34	0.47	1193	0.23
20[a]	4	2	0.01348	0.34	0.47	1193	0.28
9	1.63	1	-	0.34	0.47	1193	20.46
16	1.63	1	-	0.2	0.45	1100	81.33
17	1.63	1	-	0.2	0.45	1100	18.6
18[a]	1.63	1	-	0.34	0.47	1193	8.65


2D non-Newtonian power-law fluid flow
in vertical varying permeability
Test 18
α = 1.0 d = 1 - 4 mm, ω = 1.63, Q = 8.65 ml/s/m
n = 0.47, m = 0.34 Pa s^n, ρ = 1193 kg/m^3
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HYDRAULIC RESEARCH LAB
28th October 2014

2D non-Newtonian power-law fluid flow in vertically increasing permeability and porosity

Test 20

$\alpha = 1.0$, $\omega = 4$, $\gamma = 2$, $Q = 0.280$ ml/s

$n = 0.47$, $m = 0.34$ Pa s^n, $\rho = 1193$ kg/m^3

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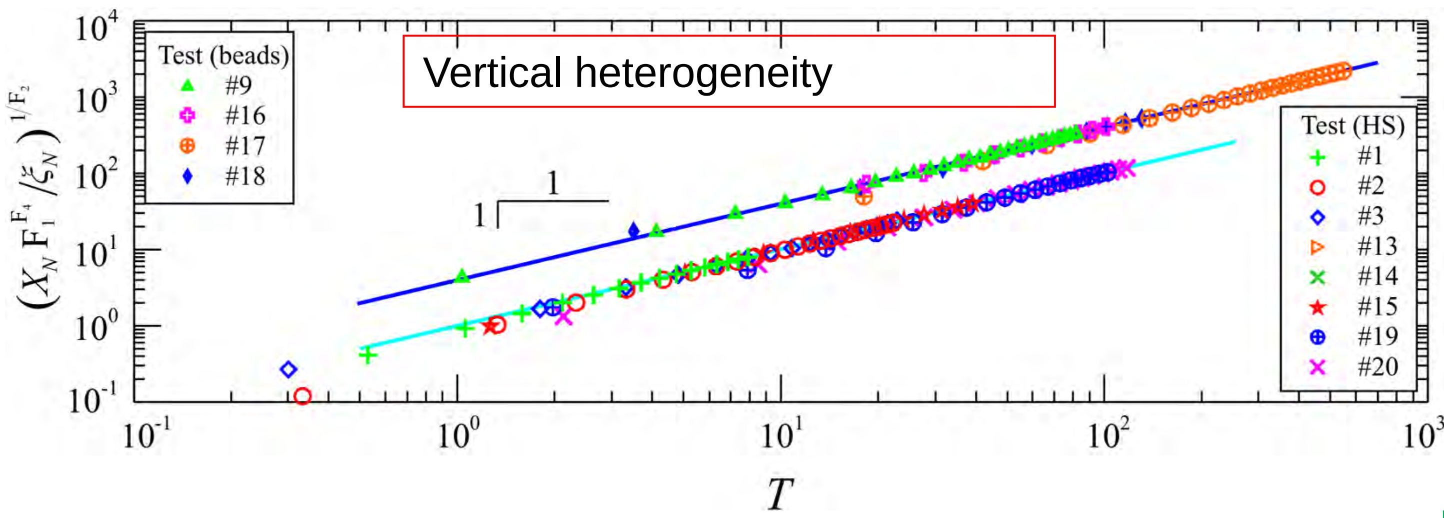
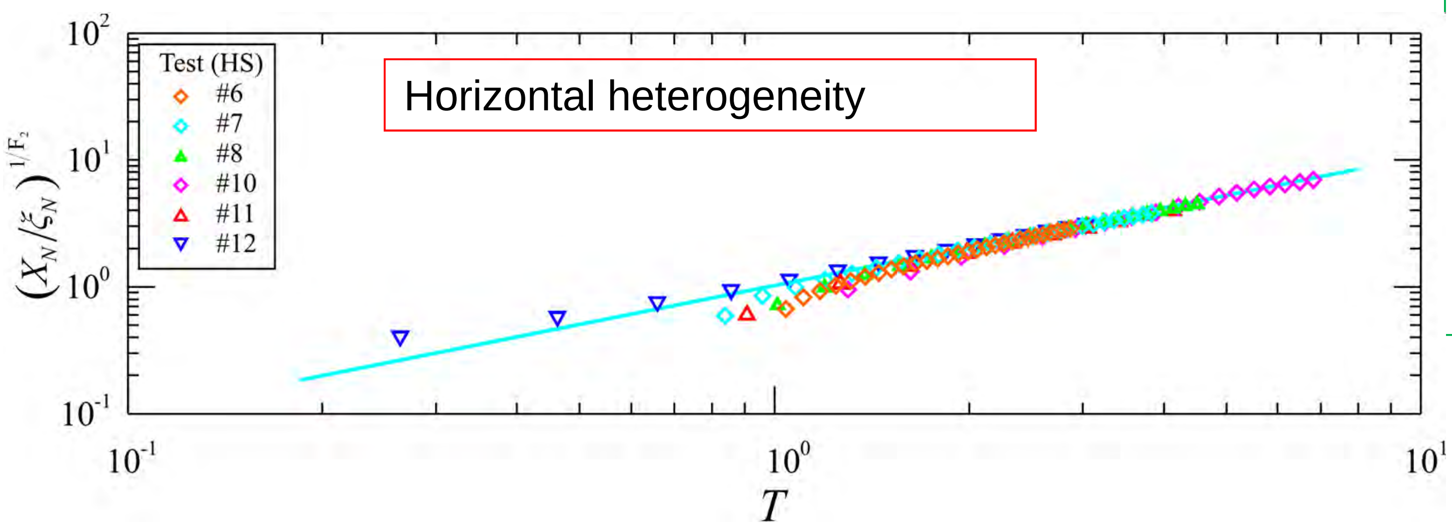
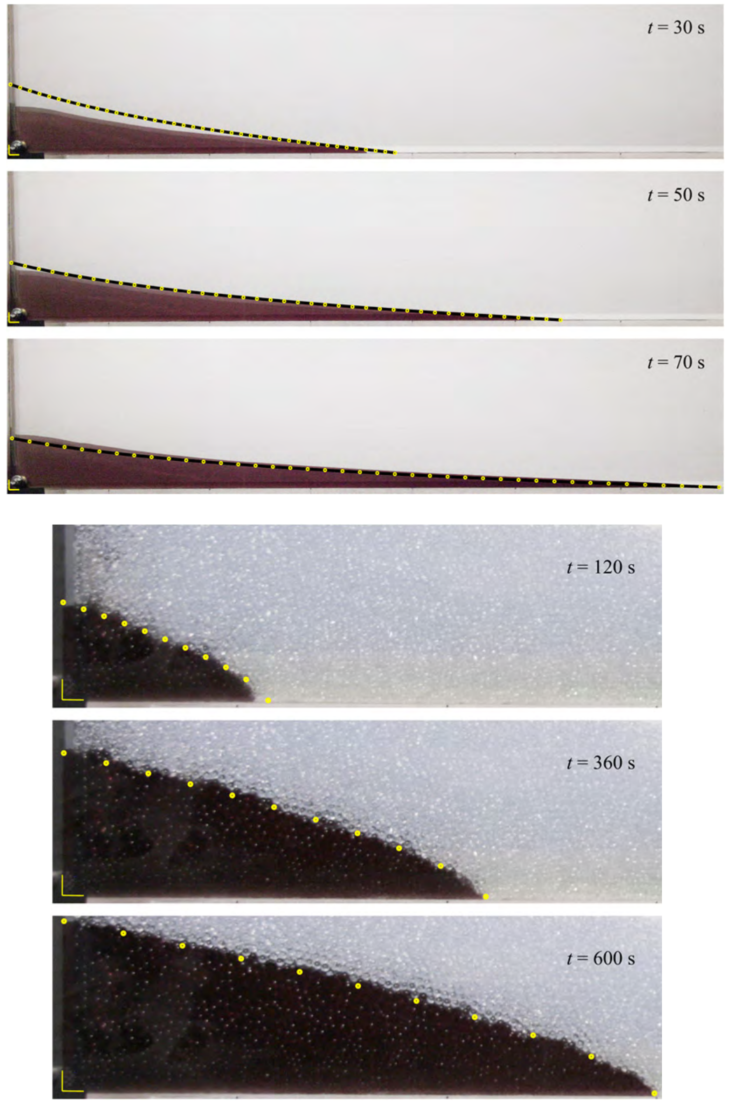
Comparison with theory: front

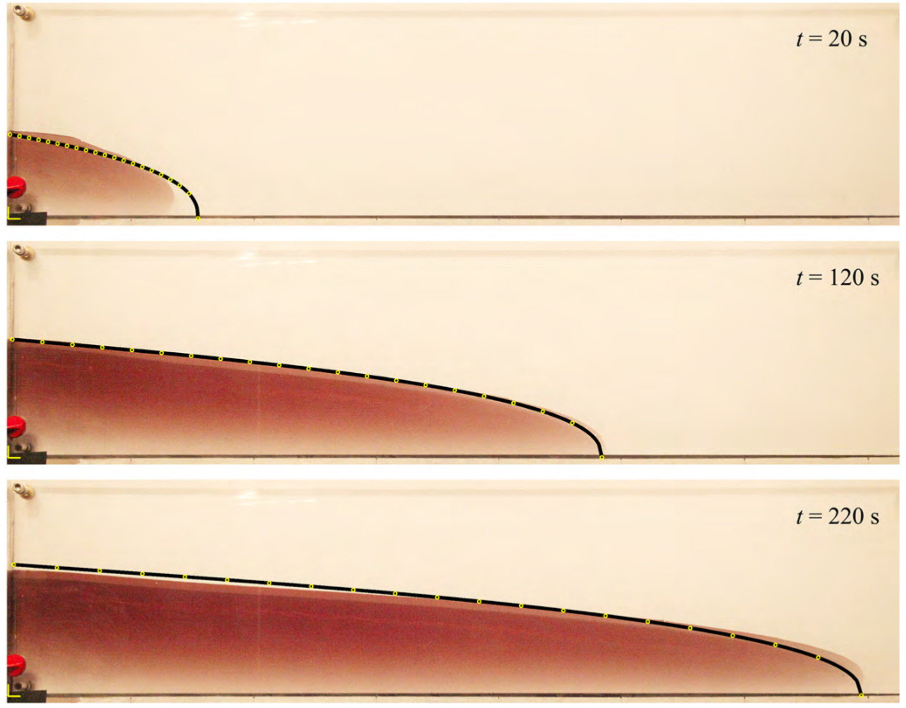




- Scaled non-dimensional position of the front end x_N against time
- Experimental results agree with theory, some discrepancy at early times

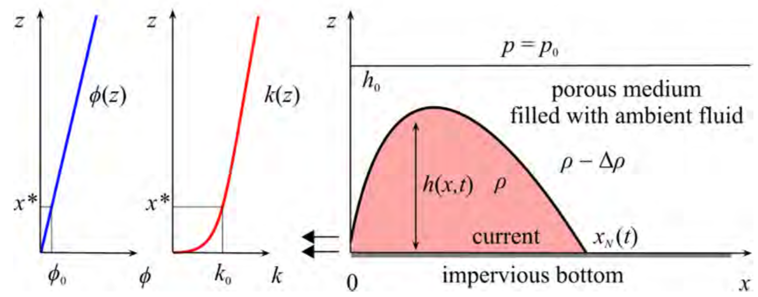
Comparison with theory: profile

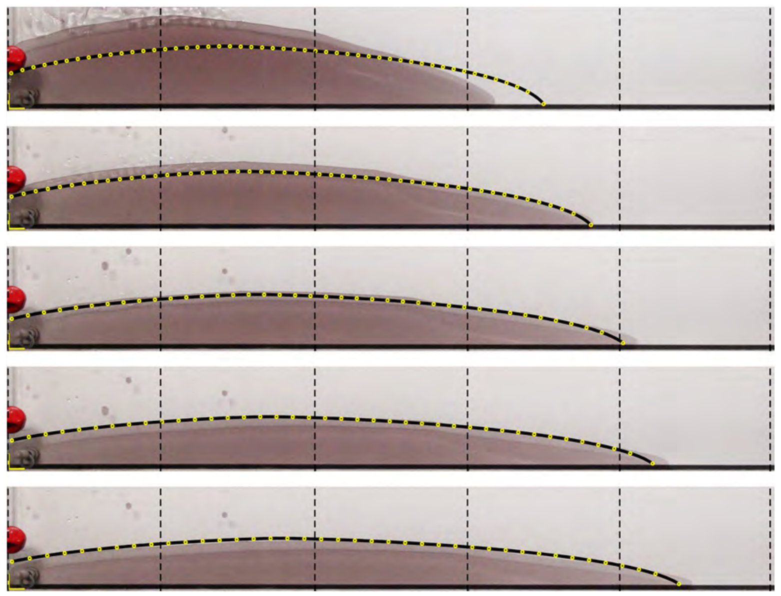


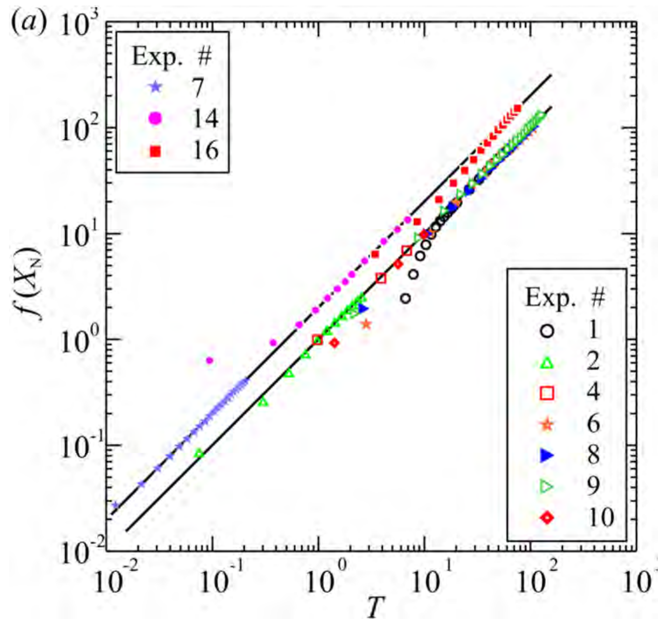


- Current profiles at different times
- Agreement fairly good, with discrepancies near the origin / at early time

Dipole flow


z
$\phi(z)$
x^*
ϕ_0
ϕ
z
$k(z)$
x^*
k_0
k
z
$p = p_0$
h_0
porous medium
filled with ambient fluid
$\rho - \Delta\rho$
$h(x,t)$ ρ
current
$x_N(t)$
impervious bottom
0
x




(a)
Exp. #
7
14
16
Exp. #
1
2
4
6
8
9
10
$f(X_N)$
T

Conclusions

- Propagation of thin GC of power-law fluids in PM amenable to similarity solution for variety of scenarios (geometry, fluid, spatial heterogeneity, release rate, …)
- Earlier results generalized (fluid, spatial heterogeneity)
- Current front/profile controlled by model parameters (α, n, γ, ω)
- Theory supported by experiments with PM simulation: i) direct ii) Hele-Shaw analogue, with different permeabilities, flow rates, and fluids
- Hele-Shaw analogy for NN flow in spatially heterogeneous PM formally developed
- Ongoing developments:
    - Confined NN flow in spatially heterogeneous PM
    - Stochastic heterogeneity